§1: Functions and change

Rmk: Most of this chapter is review from prereq.'s for our class.

§1.2: Linear functions

Def: A "linear function" is an equation of the form "$y = mx + b$" where m is the "slope" and b is the "y-int".

Rmk:
- $m > 0 \implies f$ is increasing
- $m < 0 \implies f$ is decreasing
- The equation of a line of slope m passing through (x_0, y_0) is given by

$$y - y_0 = m(x - x_0)$$

Ex: Find a line passing through $(0,2)$ and $(2,3)$.

* on your own *

§1.3: Average rate of change

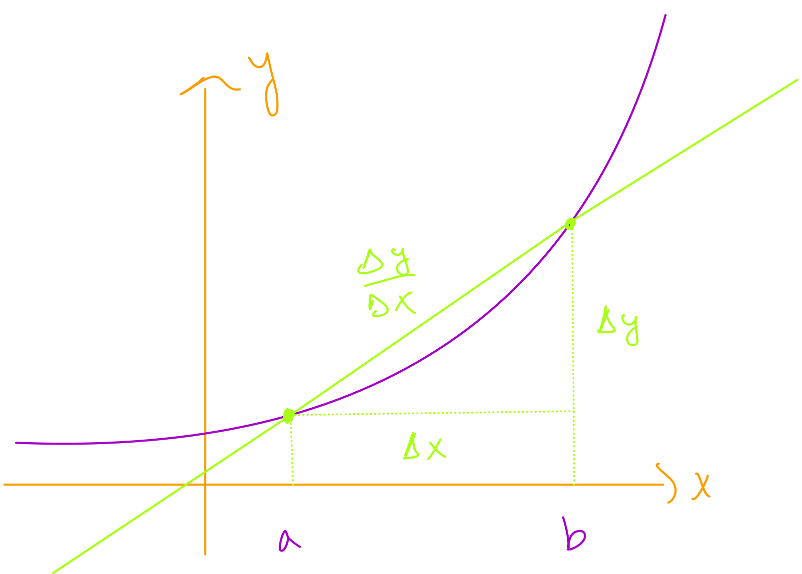
<u>Def</u>: If $f(x) = y$ is a function, then the "avg. rate of change" of f from a to b is given by

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} \qquad (b \geq a)$$

<u>Ex</u>: Compute the average rate of $y = x^2$ from $x=1$ to $x=3$.

$$\frac{\Delta y}{\Delta x} = \frac{f(3) - f(1)}{3-1} = \frac{3^2 - 1^2}{3-1} = \frac{8}{2} = 4$$

<u>Rmk</u>:



Def: • The graph of a function is "concave up" if it bends upwards as we move left to right.

• The graph of a function is "concave down" if it bends downwards as we move left to right.

Ex:

Concave up concave down

§1.5: Exponential functions

Def: A function f is said to be "exponential" if it can be written in the form

$$f(x) = k \cdot a^x$$

where $a \in \underbrace{(0,\infty)}_{\text{pos. reals}}$ and $k \in \underbrace{(-\infty,0) \cup (0,\infty)}_{\text{nonzero reals}}$.

Rmk: If $f(t) = k a^t$, then

- k is called the "initial quantity"
- $a > 1$, we say f has "exponential growth"
- $a \in (0,1)$, we say f has "exponential decay"

Ex: • $f(x) = 2^x$ growth, $2 > 1$ • $g(x) = 3 \cdot 6^x$ growth

• $h(x) = \frac{1}{2} \cdot \left(\frac{1}{3}\right)^x$ decay • $i(x) = \frac{1}{3} \cdot \left(\frac{3}{2}\right)^x$ growth

* Rules for exp. *

§1.6: Natural logarithm

Rmk: "e" denotes Euler's constant, it is a number in $(0,\infty)$ (i.e. like π)

Def: Let $x \in (0,\infty)$. The "ntrl. log." of x is the number c such that

$$\ln(x) = c \quad \text{iff} \quad e^c = x.$$

i.e $\ln(2) = c$ iff $e^c = 2$

Rmk: • $\ln(x)$ is the inverse function of e^x.

• $\ln(x)$ is not defined on $(-\infty, 0]$

Thm: Let $A, B, p \in (0,\infty)$, and suppose $B \neq 0$.

• $\ln(AB) = \ln(A) + \ln(B)$

• $\ln\left(\frac{A}{B}\right) = \ln(A) - \ln(B)$

• $\ln(A^p) = p\ln(A)$

• $\ln(e^x) = x$

• $e^{\ln(x)} = x$

• $\ln(1) = 0 \quad (e^0 = 1)$

• $\ln(e) = 1 \quad (e^1 = e)$

Ex: Expand the following

$$\ln\left(\frac{x^2y^3z}{w^2u}\right) = \ln(x^2y^3z) - \ln(w^2u)$$

$= \ln(x^2) + \ln(y^3) + \ln(z)$

$- \left(\ln(w^2) + \ln(u) \right)$

$= 2\ln(x) + 3\ln(y) + \ln(z)$

$- 2\ln(w) - \ln(u)$

§1.8: New functions from old

Ex: If $f(t) = t^2$ and $g(t) = t+2$, find

- $f(t+1)$
- $f(t) \cdot g(t)$
- $f(t) + g(t)$
- $f(g(t))$

$f(t+1) = (t+1)^2 = t^2 + 2t + 1$

$f(t)g(t) = t^2(t+2) = t^3 + 2t^2$

$f(g(t)) = f(t+2) = (t+2)^2 = \ldots$

Extra: $g(f(t)) = g(t^2) = t^2 + 2$

Rmk: Let $f(x)=y$ be a function. Choose $c \in (-\infty, \infty)$.

- The graph of $c \cdot f(x)$ is that of $f(x)$, but stretched vertically if $c > 1$, shrunk vertically if $c \in (0,1)$, and reflected about the x-axis & stretch or shrink if $c \in (-\infty, 0)$.
- The graph of $f(x) + c$ is that of $f(x)$ shifted by c units vertically
- The graph of $f(x+c)$ is that of $f(x)$ shifted by c units horizontally (right if $c < 0$ and left if $c > 0$)

Problems for §1.8:

8: $f(x) = \sqrt{x+4}$, $g(x) = x^2$

9: $f(x) = e^x$, $g(x) = x^2+1$

10: $f(x) = \frac{1}{x}$, $g(x) = 3x+4$

Find $f(f(x))$, $g(f(x))$, $f(g(x))$, and $g(g(x))$.

8: $f(x) = \sqrt{x+4}$, $g(x) = x^2$

$$f(f(x)) = f(\sqrt{x+4})$$
$$= \sqrt{\sqrt{x+4} + 4}$$

$$g(f(x)) = g(\sqrt{x+4})$$
$$= (\sqrt{x+4})^2$$
$$= x+4 \qquad ((\sqrt{w})^2 = w)$$

$$f(g(x)) = f(x^2)$$

$= \sqrt{x^2+4}$ $\quad (\sqrt{a+b} \neq \sqrt{a} + \sqrt{b})$

$g(g(x)) = g(x^2)$

$= (x^2)^2$

$= x^4$

Rmk. for (*): $a, b \in (-\infty, \infty)$, $b \neq 0$

$x^a x^b = x^{a+b}$ $\quad (**)$

$(x^a)^b = x^{ab}$

$\frac{x^a}{x^b} = x^{a-b}$

$x^{-b} = \frac{1}{x^b}$

Q: $f(x) = e^x$, $g(x) = x^2+1$

$f(g(x)) = f(\underbrace{x^2+1})$

$$= e^{x^2+1}$$

$$= e^{x^2} \cdot e^{1}$$

$$= e^{x^2} \cdot e$$

$$g(g(x)) = g(x^2+1)$$

$$= (x^2+1)^2 + 1$$

$$= x^4 + 2x^2 + 1 + 1$$